SU(3) Flavor Symmetry and CP Violating Rate Differences for Charmless $B \to PV$ Decays

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Abstract

We derive several relations between CP violating rate differences $\Delta(B \to PV) = \Gamma(B \to PV) - \Gamma(\bar{B} \to \bar{P}\bar{V})$ for charmless $B \to PV$ decays in the Standard Model using SU(3) flavor symmetry. It is found that although the relations between branching ratios of $\Delta S = 0$ and $\Delta S = -1$ processes are complicated, there are simple relations independent of hadronic models between some of the $\Delta S = 0$ and $\Delta S = -1$ rate differences due to the unitarity property of the Kobayashi-Maskawa matrix, such as $\Delta(B \to \pi^+\rho^-) = -\Delta(B \to \pi^+ K^{*-})$, $\Delta(B \to \pi^-\rho^+) = -\Delta(K^-\rho^+)$. SU(3) breaking effects are also estimated using factorization approximation. These relations can be tested at B factories in the near future.

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I. INTRODUCTION

Several charmless two body decay modes of $B_{u,d}$ mesons have been observed at CLEO [1,2]. These data have provided interesting information about the Standard Model (SM) [3–7]. With increased luminosities for B-factories at CLEO, KEK and SLAC, more useful information about charmless $B_{u,d}$ decays will be obtained. The SM can then be tested in more details. At present only some weak upper limits on the branching ratios have been obtained [8] for charmless B_s decays. However, more data on B_s decays will become available from hadron colliders, such as CDF, D0, HERAb, BTeV and LHCb in the future. These data will help to further test the SM [9]. Theoretical predictions, on the other hand, are limited by our inability to reliably calculate hadronic matrix elements related to B decays although progresses have been made in recent years [10]. In the lack of reliable calculations, attempts have been made to extract useful information from symmetry considerations. $SU(3)$ flavor symmetry [11–13] is one of the symmetries which has attracted a lot of attentions recently. It has been shown that using SU(3) symmetry it is possible to constrain [7,14] and to determine [6] one of the fundamental parameters γ in the SM for CP violation by measuring several charmless hadronic B meson decay modes. SU(3) flavor symmetry also predicts many interesting relations between CP violating observables in the SM, such as rate differences between $\Delta S = 0$ and $\Delta S = -1$ B decays. These relations will provide important test for the SM.

CP violating rate differences $\Delta(B \to a_1 a_2) = \Gamma(\bar{B} \to a_1 a_2) - \Gamma(B \to \bar{a}_1 \bar{a}_2)$ for B decays have been studied before [15–17]. Here a_i can be one of the octet pseudo-scalars P or one of the octet vector mesons V, respectively. A complete study for the charmless $B \to PP$ case was carried out in Ref. [16]. For $B \to VV$, as long as rate differences are concerned, the situation is similar to $B \to PP$. For $B \to PV$, the situation is more complicated because there are more independent SU(3) invariant decay amplitudes and more relations exist for these decays. In this paper we will concentrate on CP violating rate difference relations for $B \to PV$ decays in the SM using SU(3) flavor symmetry. $B \to PV$ decays using SU(3) flavor symmetry have also been studied in the literature recently in Ref. [18,19] with different emphases. Keeping the leading order contributions, Ref. [18] studied available data from CLEO [1,2] and obtained information about the phase angle γ and proposed new tests for the SM. Keeping all contributions Ref. [19] studied electroweak penguin effects and information about γ. In our analysis we also keep all contributions, but with the emphasis on possible relations between rate differences which have not been studied before in the literature for B to PV decays. At present CLEO collaboration has measured several $B \to PV$ modes. In the near future, B factories will measure many more of these decays with more precision and rate differences may be measured. CP violation in the SM will be tested using $B \to PV$ decays.

The paper is arranged as following. In section II we present the SU(3) decay amplitudes for charmless $B \to PV$ decays. In section III we study relations between rate differences for various B to PV decays. In section IV, we estimate SU(3) breaking effects on the relations obtained. And in section V we draw our conclusions.

II. $SU(3)$ DECAY AMPLITUDES FOR CHARMLESS $B \to PV$

The quark level effective Hamiltonian up to one loop level in electroweak interaction for charmless hadronic B decays, including the QCD corrections to the matrix elements, can be written as

$$H_{eff}^q = \frac{G_F}{\sqrt{2}}[V_{ub}V_{uq}^*(c_1 O_1 + c_2 O_2) - \sum_{i=3}^{12}(V_{ub}V_{uq}^* c_i^{uc} + V_{tb}V_{tq}^* c_i^{tc})O_i]. \tag{1}$$

The coefficients $c_{1,2}$ and $c_i^{jk} = c_i^j - c_i^k$, with j indicates the internal quark, are the Wilson Coefficients (WC). These WC's have been evaluated by several groups [20], with $|c_{1,2}| >> |c_i^j|$. In the above the factor $V_{cb}V_{cq}^*$ has been eliminated using the unitarity property of the KM matrix. The operators O_i are defined as [20],

$$
\begin{aligned}
&O_1 = (\bar{q}_i u_j)_{V-A}(\bar{u}_i b_j)_{V-A} , && O_2 = (\bar{q}u)_{V-A}(\bar{u}b)_{V-A} , \\
&O_{3,5} = (\bar{q}b)_{V-A} \textstyle\sum_{q'} (\bar{q}'q')_{V\mp A} , && O_{4,6} = (\bar{q}_i b_j)_{V-A} \textstyle\sum_{q'} (\bar{q}'_j q'_i)_{V\mp A} , \\
&O_{7,9} = \tfrac{3}{2}(\bar{q}b)_{V-A} \textstyle\sum_{q'} e_{q'} (\bar{q}'q')_{V\pm A} , && O_{8,10} = \tfrac{3}{2}(\bar{q}_i b_j)_{V-A} \textstyle\sum_{q'} e_{q'} (\bar{q}'_j q'_i)_{V\pm A} , \\
&O_{11} = \tfrac{g_s}{16\pi^2} \bar{q}\sigma_{\mu\nu} G^{\mu\nu}(1+\gamma_5)b , && O_{12} = \tfrac{Q_b e}{16\pi^2} \bar{q}\sigma_{\mu\nu} F^{\mu\nu}(1+\gamma_5)b.
\end{aligned} \tag{2}
$$

where $(\bar{a}b)_{V-A} = \bar{a}\gamma_\mu(1-\gamma_5)b$, $G^{\mu\nu}$ and $F^{\mu\nu}$ are the field strengths of the gluon and photon, respectively.

At the hadronic level, the decay amplitude for $B \to PV$ can be generically written as

$$
A(B \to PV) =< P\ V|H_{eff}^q|B >= V_{ub}V_{uq}^* T(q) + V_{tb}V_{tq}^* P(q) , \tag{3}
$$

where $T(q)$ contains contributions from the *tree* operators $O_{1,2}$ as well as *penguin* operators O_{3-12} due to charm and up quark loop corrections to the matrix elements, while $P(q)$ contains contributions purely from *penguin* due to top and charm quarks in loops. We would like to clarify the notation used here. The amplitude T in eq. 3 is usually called the "tree" amplitude which will also be referred to later on in the paper. One should, however, keep in mind that it contains the usual tree current-current contributions proportional to $c_{1,2}$ and also the u and c penguin contributions proportional to $c_i^u - c_i^c$ with $i = 3 - 12$ which is small due to cancellation between contributions from up and charm quarks in loops. Also, in general, it contains long distance contributions corresponding to internal u and c generated intermediate hadron states [21]. In our later analysis, we do not distinguish between the tree and the penguin contributions in the amplitude T unless specifically indicated.

The relative strength of the amplitudes T and P is predominantly determined by their corresponding WC's in the effective Hamiltonian. For $\Delta S = 0$ charmless decays, the dominant contributions are due to the tree operators $O_{1,2}$ and the penguin operators are suppressed by smaller WC's. Whereas for $\Delta S = -1$ decays, because the penguin contributions are enhanced by a factor of $V_{tb}V_{ts}^*/V_{ub}V_{us}^* \approx 55$ [8] compared with the tree contributions, penguin effects dominate the decay amplitudes. In this case the electroweak penguins can also play a very important role [22]. In all $\Delta S = 0$ and $\Delta S = -1$ processes, both the tree and penguin contributions have to be present in order to have rate differences. One should carefully keep track of all different contributions.

The operators $O_{1,2}$, $O_{3-6,11}$, and $O_{7-10,12}$ transform under SU(3) symmetry as $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, $\bar{3}$, and $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, respectively. These properties enable us to write the decay amplitudes for $B \to PV$ in only a few SU(3) invariant amplitudes.

For the $T(q)$ amplitude, for example, we have [12]

$$\begin{aligned} T(q) = & A_{\bar{3}}(T) B_i H(\bar{3})^i (V_l^k M_k^l) \\ & + C_{\bar{3}}^V(T) B_i V_k^i M_j^k H(\bar{3})^j + C_{\bar{3}}^M(T) B_i M_k^i V_j^k H(\bar{3})^j \\ & + A_6^V(T) B_i H(6)_k^{ij} V_j^l M_l^k + A_6^M(T) B_i H(6)_k^{ij} M_j^l V_l^k \\ & + C_6^V(T) B_i V_j^i H(6)_l^{jk} M_k^l + C_6^M(T) B_i M_j^i H(6)_l^{jk} V_k^l \\ & + A_{\overline{15}}^V(T) B_i H(\overline{15})_k^{ij} V_j^l M_l^k + A_{\overline{15}}^M(T) B_i H(\overline{15})_k^{ij} M_j^l V_l^k \\ & + C_{\overline{15}}^V(T) B_i V_j^i H(\overline{15})_l^{jk} M_k^l + C_{\overline{15}}^M(T) B_i M_j^i H(\overline{15})_l^{jk} V_k^l \,, \end{aligned} \tag{4}$$

where $B_i = (B_u, B_d, B_s) = (B^-, \bar{B}^0, \bar{B}_s^0)$ is a SU(3) triplet, M_i^j and V_i^j are the SU(3) pseudo-scalar and vector meson octets, respectively. In our analysis, we will also include the SU(3) singlets for both the psuedo-scalar and vector mesons, such that M and V become the U(3) nonet, to have some idea about the decay amplitudes for B decays involving these particles. The SU(3) invariant amplitudes $A_i(T)$ and $C_i(T)$ are in general complex due to final state interactions. The matrices $H(i)$ contain information about the transformation properties of the operators O_{1-12}.

For $q = d$, the non-zero entries of the matrices $H(i)$ are given by [12]

$$\begin{gathered} H(\bar{3})^2 = 1 \,, \;\; H(6)_1^{12} = H(6)_3^{23} = 1 \,, \;\; H(6)_1^{21} = H(6)_3^{32} = -1 \,, \\ H(\overline{15})_1^{12} = H(\overline{15})_1^{21} = 3 \,, \; H(\overline{15})_2^{22} = -2 \,, \; H(\overline{15})_3^{32} = H(\overline{15})_3^{23} = -1 \,. \end{gathered} \tag{5}$$

And for $q = s$, the non-zero entries are [15]

$$\begin{gathered} H(\bar{3})^3 = 1 \,, \;\; H(6)_1^{13} = H(6)_2^{32} = 1 \,, \;\; H(6)_1^{31} = H(6)_2^{23} = -1 \,, \\ H(\overline{15})_1^{13} = H(\overline{15})_1^{31} = 3 \,, \; H(\overline{15})_3^{33} = -2 \,, \; H(\overline{15})_2^{32} = H(\overline{15})_2^{23} = -1 \,. \end{gathered} \tag{6}$$

There are similar amplitudes for the penguins. We will indicate these SU(3) invariant amplitudes by $A_i(P)$ and $C_i(P)$.

The decay amplitudes can be written in terms of the SU(3) invariant amplitudes as

$$\begin{aligned} A(\bar{B} \to PV) = & V_{ub}V_{uq}^*(a_i^V A_i^V(T) + a_i^M A_i^M(T) + b_i^V C_i^V(T) + b_i^M C_i^M(T)) \\ & + V_{tb}V_{tq}^*(a_i^V A_i^V(P) + a_i^M A_i^M(P) + b_i^V C_i^V(P) + b_i^M C_i^M(P)). \end{aligned} \tag{7}$$

The index i is summed over $i = \bar{3}, 6, \overline{15}$. For $i = \bar{3}$, there is only one $A_{\bar{3}}$ amplitude. We use the convention: $A_{\bar{3}}^V = A_{\bar{3}}$, $a_{\bar{3}}^V = a_{\bar{3}}$ and $a_{\bar{3}}^M A_{\bar{3}}^M = 0$.

One can easily obtain the decay amplitudes for $B \to PP$ and $B \to VV$ from the above by replacing V_j^i by M_j^i, and M_j^i by V_j^i in the above expression, respectively. The SU(3) invariant amplitudes are then replaced by $A_i = A_i^V + A_i^M$ and $C_i = C_i^V + C_i^M$. In these two cases, due to the anti-symmetric nature in exchanging the upper two indices of $H_k^{ij}(6)$ and the symmetric structure of the two mesons in the final states, $C_6 - A_6$ always appear together [12] if the singlets in the final states are removed. Therefore in these cases there are in total 5 independent SU(3) invariant amplitudes for each case. However if singlets are included, decay modes involving singlets in the final states can be used to separate A_6 and C_6. In this case there are in total 6 independent amplitudes. For $B \to PV$, even the symmetric structure of the two mesons in the final states is lost, there are relations between A_6^i and C_6^i if the singlets η_1 and the combination of $(\sqrt{2/3})\omega + \phi/\sqrt{3}$ are absent, there are total 10 independent invariant amplitudes which agree with the analysis in Ref. [19]. When the singlets are introduced, A_i and C_i are all independent. There are total 11 of them. The analysis in this case is more complicated compared with the cases for $B \to PP$ and $B \to VV$. Expanding eq. 4, we obtain the coefficients for each individual decays. In tables 1 - 3 we list the coefficients a_i and b_i for $\Delta S = 0$ $B \to PV$ decays, and in Tables 4-6 for $\Delta S = -1$ $B \to PV$ decays.

Two remarks are in order: a) The amplitudes A_i correspond to annihilation contributions, as can be seen from eq. 4 where B_i is contracted with one of the index in $H(j)$, are small compared with the amplitudes C_i from model calculations. The smallness of these annihilation amplitudes can be tested using $B_d \to K^- K^{*+}, K^+ K^{*-}$ and $B_s \to \pi^+\rho^-, \pi^-\rho^+, \pi^0\rho^0, \pi^0\omega$ because these decays have only annihilation contributions. b) Many analyses have been carried out using SU(3) classification of quark level diagrams [13] in the literature. In most cases such analyses give the same results as the use of SU(3) invariant amplitudes discussed here. However, in some cases the classification according to quark level diagrams without care could be misleading. One should be careful to include all possible contributions to have a complete study [19]. For example, neglecting annihilation contributions, the operators $O_{1,2}$ would have vanishing contributions to $B_u \to K^- K^{*0}, \bar{K}^0 K^{*-}, \pi^- \bar{K}^{*0}, \bar{K}^0 \rho^-$, $B_d \to K^0 \bar{K}^{*0}, \bar{K}^0 K^{*0}$, and $B_s \to K^0 \bar{K}^{*0}, \bar{K}^0 K^{*0}$. However, from the Tables, we find that these decays are proportional to $C_{\bar{3}}^{V,M} - C_6^{V,M} - C_{\overline{15}}^{V,M}$. These combinations are not necessarily zero without additional assumptions. Whether naive diagram analysis is valid has to be tested experimentally. This can be achieved by measuring the decay modes mentioned in the remarks a) and b). Without evidence from experimental data, we have to keep the most general form and carry out analyses accordingly.

III. CP VIOLATING RATE DIFFERENCES FOR CHARMLESS $B \to PV$

One can find many relations between different B decays using SU(3) flavor symmetry to test the SM in different ways. A particularly interesting class of relations for $B \to PV$ is the CP violating rate difference $\Delta(B \to PV) = \Gamma(B \to PV) - \Gamma(\bar{B} \to \bar{P}\bar{V})$. Theoretical calculations of rate differences are difficult because one not only needs to calculate the short distance decay amplitudes but also the long distance contributions, especially the final state interaction phases. This is a difficult task at present. Due to this reason, even though CP violation is measured for certain B decay modes and compared with hadronic model calculations, one is not sure if the results agree with the SM predictions. In this section we will derive several relations which do not depend on the dynamics of hadronization processes using the SU(3) decay amplitudes obtained in the previous section. These relations can provide hadronization model independent tests for CP violation in the SM.

Using the SU(3) decay amplitudes obtained in the previous section, one can find that some decay amplitudes for $\Delta S = 0$ and $\Delta S = -1$, $B \to PV$ decays have the following peculiar form [15–17,23] ,

$$
\begin{aligned}
A(d) &= V_{ub}V^*_{ud}T + V_{tb}V^*_{td}P, \\
A(s) &= V_{ub}V^*_{us}T + V_{tb}V^*_{ts}P.
\end{aligned} \tag{8}
$$

Due to different KM matrix elements involved in $A(d)$ and $A(s)$, although the amplitudes have some similarities, the branching ratios are not simply related. However, when considering rate difference, the situation is dramatically different. Because a simple property of the KM matrix element [24], $Im(V_{ub}V^*_{ud}V^*_{tb}V_{td}) = -Im(V_{ub}V^*_{us}V^*_{tb}V_{ts})$, in the SU(3) limit we have

$$
\Delta(d) = -\Delta(s), \tag{9}
$$

where $\Delta(i) = (|A(i)|^2 - |\bar{A}(i)|^2)\lambda_{ab}/(8\pi m_B)$ is the CP violating rate difference defined earlier and $\lambda_{ab} = \sqrt{1 - 2(m_a^2 + m_b^2)/m_B^2 + (m_a^2 - m_b^2)^2/m_B^4}$ with $m_{a,b}$ being the masses of the two particles in the final state.

The above non-trivial equality does not depend on the numerical values of the final state re-scattering phases. Of course these relations are true only for models with three generations. Therefore they provide dynamic model independent tests for the three generation Standard Model.

We find the following equalities:

$$
\begin{aligned}
&(1)\ \ \Delta(B_u \to K^- K^{*0}) = -\Delta(B_u \to \pi^- \bar{K}^{*0})\ , \\
&(2)\ \ \Delta(B_d \to \bar{K}^0 K^{*0}) = -\Delta(B_s \to K^0 \bar{K}^{*0})\ , \\
&(3)\ \ \Delta(B_u \to K^0 K^{*-}) = -\Delta(B_u \to \bar{K}^0 \rho^-)\ , \\
&(4)\ \ \Delta(B_d \to K^0 \bar{K}^{*0}) = -\Delta(B_s \to \bar{K}^0 K^{*0})\ , \\
&(5)\ \ \Delta(B_d \to \pi^- \rho^+) = -\Delta(B_s \to K^- K^{*+})\ , \\
&(6)\ \ \Delta(B_s \to \pi^- K^{*+}) = -\Delta(B_d \to K^- \rho^+)\ ,
\end{aligned}
$$

$$
\begin{aligned}
&(7)\ \ \Delta(B_d \to \pi^+\rho^-) = -\Delta(B_s \to K^+K^{*-}) \ ,\\
&(8)\ \ \Delta(B_s \to K^+\rho^-) = -\Delta(B_d \to \pi^+K^{*-}) \ ,\\
&(9)\ \ \Delta(B_u \to \eta_1\rho^-) = -\Delta(B_u \to \eta_1 K^{*-}) \ ,\\
&(10)\ \ \Delta(B_s \to \eta_1 K^{*0}) = -\Delta(B_d \to \eta_1 \bar{K}^{*0}) \ ,\\
&(11)\ \ \Delta(B_d \to K^-K^{*+}) = -\Delta(B_s \to \pi^-\rho^+) \ ,\\
&(12)\ \ \Delta(B_d \to K^+K^{*-}) = -\Delta(B_s \to \pi^+\rho^-).
\end{aligned} \tag{10}
$$

If it turns out that the annihilation contributions are all small, as can be tested in $B_d \to K^-K^{*+}$, K^+K^{*-} and $B_s \to \pi^+\rho^-$, $\pi^-\rho^+$, $\pi^0\rho^0$, $\pi^0\omega$, there are additional relations for rate differences. We find the following equalities,

$$
\begin{aligned}
&(1) = (2) \ , \qquad &(3) = (4) \ ,\\
&(5) = (6) \ , \qquad &(7) = (8).
\end{aligned} \tag{11}
$$

IV. SU(3) BREAKING EFFECTS

The relations obtained in the previous section hold in the SU(3) limit. The SU(3) symmetry need to be tested experimentally. To this end we comment that SU(3) predictions for $B \to PV$ with charmed vector meson V can be independently tested by using $B \to D^*\pi$ and $B \to D^*K$. These decays receive contributions from the tree operators $O_{1,2}$ only. This provides a clear test for SU(3) symmetry free from penguin contaminations. In the SU(3) symmetry, the ratio of these branching ratios is equal to $r = Br(B \to D^*\pi)/Br(B \to D^*K) = |V_{ud}/V_{us}|^2$. Factorization calculation gives a SU(3) breaking factor and $r = (f_\pi^2/f_k^2)|V_{ud}/V_{us}|^2$. This can be tested experimentally. Results obtained from this will give us a good guidance on how well CP violation can be tested in charmless $B \to PV$ decays.

If SU(3) is broken these relations need to be modified. We now study how these relations are modified when SU(3) breaking effects are included. Since no reliable calculational tool exists, in the following we will use factorization approximation neglecting the annihilation contributions to estimate the SU(3) breaking effects. We find [3,9]

$$
\begin{aligned}
&(1)\ \ \Delta(B_u \to K^- K^{*0}) = -(\frac{F_1^{B\to K}(m^2_{K^*})}{F_1^{B\to\pi}(m^2_{K^*})})^2 \Delta(B_u \to \pi^- \bar{K}^{*0})\ ,\\
&(2)\ \ \Delta(B_d \to \bar{K}^0 K^{*0}) = -(\frac{F_1^{B\to K}(m^2_{K^*}}{F_1^{B_s\to K}(m^2_{K^*})})^2 \Delta(B_s \to K^0 \bar{K}^{*0})\ ,\\
&(3)\ \ \Delta(B_u \to K^0 K^{*-}) = -(\frac{A_0^{B\to K^*}(m^2_K)}{A_0^{B\to\rho}(m^2_\rho)})^2 \Delta(B_u \to \bar{K}^0 \rho^-)\ ,\\
&(4)\ \ \Delta(B_d \to K^0 \bar{K}^{*0}) = -(\frac{A_0^{B\to K^*}(m^2_K)}{A_0^{B_s\to K^*}(m^2_K)})^2 \Delta(B_s \to \bar{K}^0 K^{*0})\ ,\\
&(5)\ \ \Delta(B_d \to \pi^- \rho^+) = -(\frac{f_\pi A_0^{B\to\rho}(m^2_\pi)}{f_K A_0^{B_s\to K^*}(m^2_K)})^2 \Delta(B_s \to K^- K^{*+})\ ,\\
&(6)\ \ \Delta(B_s \to \pi^- K^{*+}) = -(\frac{f_\pi A_0^{B_s\to K^*}(m^2_\pi)}{f_K A_0^{B\to\rho}(m^2_K)})^2 \Delta(B_d \to K^- \rho^+)\ ,\\
&(7)\ \ \Delta(B_d \to \pi^+ \rho^-) = -(\frac{f_\rho F_1^{B\to\pi}(m^2_\rho)}{f_{K^*} F_1^{B_s\to K}(m^2_{K^*}})^2 \Delta(B_s \to K^+ K^{*-})\ ,\\
&(8)\ \ \Delta(B_s \to K^+ \rho^-) = -(\frac{f_\rho F_1^{B_s\to K}(m^2_\rho)}{f_{K^*} F_1^{B\to\pi}(m^2_{K^*})})^2 \Delta(B_d \to \pi^+ K^{*-}).
\end{aligned}
\tag{12}
$$

In the above we have not listed the corrections for the equalities (9), (10), (11) and (12). The corrections to the decay modes in (9) and (10) involving η_1 are complicated because there are two corrections to the amplitudes and also because mixings between η_1 and η_8. It is difficult to use these decay modes to have clear tests. The decay modes in (11) and (12) are all pure annihilation type of decays which are zero in the naive factorization approximation and likely to be small. We will not discuss these decay modes further here.

The decay modes in (1), (2), (3) and (4) do not have contributions from $O_{1,2}$ in the factorization approximation. Therefore their rate differences are expected to be very small. However one should be careful in drawing this conclusion, as already mentioned earlier, due to the fact that the vanishing contributions from $O_{1,2}$ to these modes is not a SU(3) prediction even when annihilation contributions are neglected. It may happen that the rate differences are larger than expected. We have to wait experimental data to tell us more.

At e^+e^- B factories the best chances to test the above listed relations are from (5), (6), (7) and (8). To a good approximation, we have

$$
\begin{aligned}
&(a) \quad \Delta(B_d \to \pi^- \rho^+) \approx -(\frac{f_\pi A_0^{B\to\rho}(m_\pi^2)}{f_K A_0^{B\to\rho}(m_K^2)})^2 \Delta(B_d \to K^- \rho^+)\ ,\\
&(b) \quad \Delta(B_d \to \pi^+ \rho^-) \approx -(\frac{f_\rho F_1^{B\to\pi}(m_\rho^2)}{f_{K^*} F_1^{B\to\pi}(m_{K^*}^2)})^2 \Delta(B_d \to \pi^+ K^{*-})\ . \qquad (13)
\end{aligned}
$$

The ratios $A^{B\to\rho}(m_\pi^2)/A_0^{B\to\rho}(m_K^2)$ and $F_1^{B\to\pi}(m_\rho^2)/F_1^{B\to\pi}(m_{K^*}^2)$ are all close to one[3]. The corrections are dominated by the decay constants f_i. The relation (a) has, then, a larger correction than (b) because $f_\rho/f_{K^*} \approx 1$ and $f_\pi/f_K \approx 0.82$.

The decay branching ratios for $B_d \to \pi^+ K^{*-}$ and the combined $B_d \to \pi^+\rho^-,\ \pi^-\rho^+$ have been measured at CLEO [2] with $Br(B_d \to \pi^+ K^{*-}) = (22^{+8+4}_{-6-5}) \times 10^{-6}$, and $Br(B_d \to \pi^+\rho^- + \pi^-\rho^+) = (35^{+11}_{-10} \pm 5) \times 10^{-6}$, respectively. There is only an upper bound for $Br(B_d \to K^-\rho^+) < 25 \times 10^{-6}$ at the 90% c.l.. If we normalize the rate differences by a common branching ratio, for example, $Br(B_d \to \pi^+ K^{*-})$, the normalized rate differences for $B_d \to \pi^-\rho^+$, $K^-\rho^+$ are of order a few percent [3], while for $B_d \to \pi^+\rho^-$, $\pi^+ K^{*-}$ are of order 20% [3]. These can be measured at B factories with good precision. The relations predicted can be tested. Even earlier a test can be performed with the combined data

$$
\Delta(B_d \to \pi^-\rho^+) + \Delta(B_d \to \pi^+\rho^-) \approx -(\frac{f_\pi}{f_K})^2 \Delta(B_d \to K^-\rho^+) - (\frac{f_\rho}{f_{K^*}})^2 \Delta(B_d \to \pi^+ K^{*-})\ .
$$

When B_s decay modes are measured, such as at CDF, D0, BTeV, HERAb, and LHCb all modes in (5), (6), (7) and (8) can provide good tests for the Standard Model.

V. CONCLUSIONS AND DISCUSSIONS

We would like to point out that the relations obtained for the CP violating rate differences also hold for any three generation models in which flavor changing and CP violating source is solely due to KM matrix, such as multi-Higgs doublet models with neutral flavor current conservation by exchanging of Higgs bosons at tree level and model with anomalous three gauge boson couplings as new physics source.

To conclude we find that although the relations between branching ratios of $\Delta S = 0$ and $\Delta S = -1$ processes are complicated, there are simple relations between some of the

$\Delta S = 0$ and $\Delta S = -1$ rate differences due to the unitarity property of the KM matrix, such as $\Delta(B \to \pi^+\rho^-) = -\Delta(B \to \pi^+ K^{*-})$, $\Delta(B \to \pi^-\rho^+) = -\Delta(K^-\rho^+)$. We also estimated SU(3) breaking effects using factorization approximation. The relations involving B_d and B_u decays can soon be tested by CLEO, Babar and Belle collaborations. While the relations involving B_s decays can be carried out at hadron colliders, such as CDF, D0, BTeV, HERAb and LHCb. CP violation can be tested in a model independent way for the Standard Model using the relations derived in this paper soon. We emphasis that the importance of these tests is their independence from hadronic models. We urge our experimental colleagues to carry out such analyses.

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Table 1. SU(3) decay amplitudes for $\Delta S = 0$, $B_u \to PV$ decays.

Decay Mode		$a_{\bar{3}}$	a_6^V	a_6^M	$a_{\overline{15}}^V$	$a_{\overline{15}}^M$	$b_{\bar{3}}^V$	$b_{\bar{3}}^M$	b_6^V	b_6^M	$b_{\overline{15}}^V$	$b_{\overline{15}}^M$
$B_u \to K^- K^{*0}$		0	1	0	3	0	0	1	0	-1	0	-1
$B_u \to K^0 K^{*-}$		0	0	1	0	3	1	0	-1	0	-1	0
$B_u \to \eta_8 \rho^-$	$\frac{1}{\sqrt{6}}$	0	1	1	3	3	1	1	-3	1	3	3
$B_u \to \eta_1 \rho^-$	$\frac{1}{\sqrt{3}}$	0	1	1	3	3	1	1	0	1	0	3
$B_u \to \pi^0 \rho^-$	$\frac{1}{\sqrt{2}}$	0	1	-1	3	-3	-1	1	-1	1	5	3
$B_u \to \pi^- \phi$		0	0	0	0	0	0	0	0	1	0	-1
$B_u \to \pi^- \rho^0$	$\frac{1}{\sqrt{2}}$	0	-1	1	-3	3	1	-1	1	-1	3	5
$B_u \to \pi^- \omega$	$\frac{1}{\sqrt{2}}$	0	1	1	3	3	1	1	1	-1	3	1

Table 2. SU(3) decay amplitudes for $\Delta S = 0$, $B_d \to PV$ decays.

Decay Mode			$a_{\bar{3}}$	a_6^V	a_6^M	$a_{\overline{15}}^V$	$a_{\overline{15}}^M$	$b_{\bar{3}}^V$	$b_{\bar{3}}^M$	b_6^V	b_6^M	$b_{\overline{15}}^V$	$b_{\overline{15}}^M$	
$B_d \to K^0 \bar{K}^{*0}$		(	1	1	0	-1	-2	1	0	-1	0	-1	0	)
$B_d \to \bar{K}^0 K^{*0}$		(	1	0	1	-2	-1	0	1	0	-1	0	-1	)
$B_d \to K^- K^{*+}$		(	1	-1	1	3	-1	0	0	0	0	0	0	)
$B_d \to \pi^- \rho^+$		(	1	-1	0	3	-2	1	0	1	0	3	0	)
$B_d \to \pi^+ \rho^-$		(	1	0	-1	-2	3	0	1	0	1	0	3	)
$B_d \to K^+ K^{*-}$		(	1	1	-1	-1	3	0	0	0	0	0	0	)
$B_d \to \eta_8 \phi$	$\frac{1}{\sqrt{6}}$	(	-2	-2	-2	2	2	0	0	0	1	0	-1	)
$B_d \to \eta_8 \omega$	$\frac{1}{2\sqrt{3}}$	(	2	-1	-1	1	1	1	1	-3	-1	3	1	)
$B_d \to \eta_8 \rho^0$	$\frac{1}{2\sqrt{3}}$	(	0	-1	-1	5	5	-1	-1	3	-1	-3	5	)
$B_d \to \eta_1 \phi$	$\frac{1}{\sqrt{3}}$	(	1	1	1	-1	-1	0	0	0	1	0	-1	)
$B_d \to \eta_1 \omega$	$\frac{1}{\sqrt{6}}$	(	2	-1	-1	1	1	1	1	0	-1	0	1	)
$B_d \to \eta_1 \rho^0$	$\frac{1}{\sqrt{6}}$	(	0	-1	-1	5	5	-1	-1	0	-1	0	5	)
$B_d \to \pi^0 \rho^0$	$\frac{1}{2}$	(	2	-1	-1	1	1	1	1	1	1	-5	-5	)
$B_d \to \pi^0 \phi$	$\frac{1}{\sqrt{2}}$	(	0	0	0	0	0	0	0	0	-1	0	1	)
$B_d \to \pi^0 \omega$	$\frac{1}{2}$	(	0	-1	-1	5	5	-1	-1	-1	1	5	-1	)

Table 3. SU(3) decay amplitudes for $\Delta S = 0$, $B_s \to PV$ decays.

Decay Mode		$a_{\bar{3}}$	a_6^V	a_6^M	$a_{\overline{15}}^V$	$a_{\overline{15}}^M$	$b_{\bar{3}}^V$	$b_{\bar{3}}^M$	b_6^V	b_6^M	$b_{\overline{15}}^V$	$b_{\overline{15}}^M$
$B_s \to K^0\phi$		0	0	-1	0	-1	1	0	-1	1	-1	-1
$B_s \to K^0\rho^0$	$\frac{1}{\sqrt{2}}$	0	1	0	1	0	0	-1	0	-1	0	5
$B_s \to K^0\omega$	$\frac{1}{\sqrt{2}}$	0	-1	0	-1	0	0	1	0	-1	0	1
$B_s \to K^+\rho^-$		0	-1	0	-1	0	0	1	0	1	0	3
$B_s \to \pi^-K^{*+}$		0	0	-1	0	-1	1	0	1	0	3	0
$B_s \to \pi^0K^{*0}$	$\frac{1}{\sqrt{2}}$	0	0	1	0	1	-1	0	-1	0	5	0
$B_s \to \eta_1K^{*0}$	$\frac{1}{\sqrt{3}}$	0	-1	-1	-1	-1	1	1	0	-1	0	-1
$B_s \to \eta_8K^{*0}$	$\frac{1}{\sqrt{6}}$	0	2	-1	2	-1	1	-2	-3	2	3	2

Table 4. SU(3) decay amplitudes for $\Delta S = -1$, $B_u \to PV$ decays.

Decay Mode		$a_{\bar{3}}$	a_6^V	a_6^M	$a_{\overline{15}}^V$	$a_{\overline{15}}^M$	$b_{\bar{3}}^V$	$b_{\bar{3}}^M$	b_6^V	b_6^M	$b_{\overline{15}}^V$	$b_{\overline{15}}^M$
$B_u \to \pi^- \bar{K}^{*0}$		0	1	0	3	0	0	1	0	-1	0	-1
$B_u \to \eta_8 K^{*-}$	$\frac{1}{\sqrt{6}}$	0	1	-2	3	-6	-2	1	0	1	6	3
$B_u \to \eta_1 K^{*-}$	$\frac{1}{\sqrt{3}}$	0	1	1	3	3	1	1	0	1	0	3
$B_u \to K^- \phi$		0	1	0	3	0	0	1	0	0	0	-2
$B_u \to K^- \rho^0$	$\frac{1}{\sqrt{2}}$	0	0	1	0	3	1	0	1	-2	3	4
$B_u \to K^- \omega$	$\frac{1}{\sqrt{2}}$	0	0	1	0	3	1	0	1	0	3	2
$B_u \to \bar{K}^0 \rho^-$	1	0	0	1	0	3	1	0	-1	0	-1	0
$B_u \to \pi^0 K^{*-}$	$\frac{1}{\sqrt{2}}$	0	1	0	3	0	0	1	-2	1	4	3

Table 5. SU(3) decay amplitudes for $\Delta S = -1$, $B_d \to PV$ decays.

Decay Mode		$a_{\bar{3}}$	a_6^V	a_6^M	$a_{\overline{15}}^V$	$a_{\overline{15}}^M$	$b_{\bar{3}}^V$	$b_{\bar{3}}^M$	b_6^V	b_6^M	$b_{\overline{15}}^V$	$b_{\overline{15}}^M$
$B_d \to \bar{K}^0 \rho^0$	$\frac{1}{\sqrt{2}}$	0	0	1	0	1	-1	0	1	-2	1	4
$B_d \to \bar{K}^0 \omega$	$\frac{1}{\sqrt{2}}$	0	0	-1	0	-1	1	0	-1	0	-1	2
$B_d \to \bar{K}^0 \phi$		0	-1	0	-1	0	0	1	0	0	0	-2
$B_d \to \pi^+ \bar{K}^{*-}$		0	-1	0	-1	0	0	1	0	1	0	3
$B_d \to K^- \rho^+$		0	0	-1	0	-1	1	0	1	0	3	0
$B_d \to \pi^0 \bar{K}^{*0}$	$\frac{1}{\sqrt{2}}$	0	1	0	1	0	0	-1	-2	1	4	1
$B_d \to \eta_1 \bar{K}^{*0}$	$\frac{1}{\sqrt{3}}$	0	-1	-1	-1	-1	1	1	0	-1	0	-1
$B_d \to \eta_8 \bar{K}^{*0}$	$\frac{1}{\sqrt{6}}$	0	-1	2	-1	2	-2	1	0	-1	6	-1

Table 6. SU(3) decay amplitudes for $\Delta S = -1$, $B_s \to PV$ decays.

Decay Mode		$a_{\bar{3}}$	a_6^V	a_6^M	$a_{\overline{15}}^V$	$a_{\overline{15}}^M$	$b_{\bar{3}}^V$	$b_{\bar{3}}^M$	b_6^V	b_6^M	$b_{\overline{15}}^V$	$b_{\overline{15}}^M$
$B_s \to K^0 \bar{K}^{*0}$		1	0	1	-2	-1	0	1	0	-1	0	-1
$B_s \to \bar{K}^0 K^{*0}$		1	1	0	-1	-2	1	0	-1	0	-1	0
$B_s \to K^- K^{*+}$		1	-1	0	3	-2	1	0	1	0	3	0
$B_s \to \pi^- \rho^+$		1	-1	1	3	-1	0	0	0	0	0	0
$B_s \to \pi^+ \rho^-$		1	1	-1	-1	3	0	0	0	0	0	0
$B_s \to K^+ K^{*-}$		1	0	-1	-2	3	0	1	0	1	0	3
$B_s \to \eta_8 \phi$	$\sqrt{\frac{2}{3}}$	-1	0	0	2	2	-1	-1	0	0	3	2
$B_s \to \eta_8 \rho^0$	$\frac{1}{\sqrt{3}}$	0	-1	-1	2	2	0	0	0	2	0	-4
$B_s \to \eta_8 \omega$	$\frac{1}{\sqrt{3}}$	1	0	0	1	1	0	0	0	0	0	-2
$B_s \to \eta_1 \phi$	$\frac{1}{\sqrt{3}}$	1	0	0	-2	-2	1	1	0	0	0	-2
$B_s \to \eta_1 \omega$	$\sqrt{\frac{2}{3}}$	1	0	0	1	1	0	0	0	0	0	1
$B_s \to \eta_1 \rho^0$	$\sqrt{\frac{2}{3}}$	0	-1	-1	2	2	0	0	0	-1	0	2
$B_s \to \pi^0 \rho^0$		1	0	0	1	1	0	0	0	0	0	0
$B_s \to \pi^0 \phi$	$\sqrt{2}$	0	0	0	0	0	0	0	-1	0	2	0
$B_s \to \pi^0 \omega$		0	-1	-1	2	2	0	0	0	0	0	0